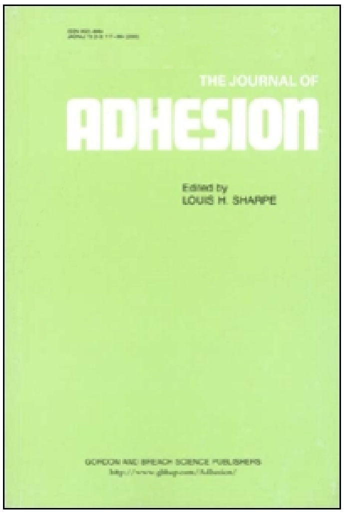

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Free Vibration Analysis and Design of an Adhesively Bonded Corner Joint with Double Support

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This study carries out the three dimensional free vibration analysis of an adhesively bonded corner joint and investigates the effect of an additional horizontal support to the adhesive corner joint with single support on the first ten natural frequencies and mode shapes. In the presence of a horizontal support the effects of the vertical support length, the adhesive thickness, the plate thickness, and the joint length on the natural frequencies and modal strain energies of the adhesive joint were also investigated using the back-propagation Artificial Neural Network (ANN) method and the finite element method. The natural frequencies and modal strain energies increased with increasing plate thickness, whereas an adverse effect was observed for increasing joint length. Both horizontal and vertical support lengths exhibited similar effects but the adhesive thickness had a negligible effect. The plate thickness and the joint length are dominant geometrical parameters in comparison with both horizontal and vertical support lengths. The proposed ANN models were combined with the Genetic Algorithm in order to determine the optimal corner joint in which the maximum natural frequency and minimum elastic modal strain energy are achieved for each natural frequency and mode shape of the adhesive corner joint and the optimal dimensions were given versus one geometrical parameter.

Keywords: Adhesive; Artificial neural network; Corner joint; Free vibration; Genetic algorithm; Mode shape; Natural frequency; Optimisation

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1. INTRODUCTION

The adhesive bonding technique allows designers to join similar and dissimilar materials as a result of recent developments in adhesive technology, very strong adhesives are available which can sustain plastic deformations under high static and dynamic loadings [1–3]. Today's applications require that adhesive joints serve under serious dynamic loads as well as the static loads. The vibration characteristics of adhesive joints, such as natural frequencies, mode shapes, and modal strain energies affect their fatigue life since the harmonic loads and short period-impact loads may cause structural adhesive joints to vibrate constantly or randomly. The free and forced vibrations of adhesively bonded stiffened composites or metal beams and plates have been investigated by the researchers.

Saito and Tani [4] showed that the overlap length and the mode number affected the natural frequencies and logarithmic decrement of the vibration of a viscoelastic adhesive single lap joint. He and Rao [5–6] modeled the coupled transverse and longitudinal vibration of a viscoelastic bonded lap joint, considering both shear and thickness deformation in the adhesive layer, and presented numerical solutions for free vibration of the adhesive joint yielding the natural frequencies, loss factors and mode shapes. Rao and Zhou [7] investigated the effects of the structural parameters and mechanical properties of the adhesive layer on the systems modal loss factors and resonance frequencies for the transverse vibration and damping of an adhesively bonded tubular lap joint under a fixed-fixed boundary condition. They assumed that both the shear and bending deformations of the adhesive layer contributed to the energy dissipated by the adhesive joint.

Ko *et al.* [8] determined the natural frequencies and corresponding mode shapes of laminated lap joints with various overlap lengths and plate thicknesses using an isoparametric adhesive interface element and a first order shear deformation plate element. Lin and Ko [9] investigated the effects of various patch sizes and adhesive modulus on the free vibration characteristics of a patched composite plate. He and Oyadiji [10] found that the Young's modulus and Poisson's ratio of the adhesive material had a minor effect on the transverse natural frequencies and mode shapes of an adhesively bonded cantilevered single lap joint, and the even mode shapes were affected considerably by the adhesive strength. Kaya *et al*. [11] also investigated the effects of various parameters on the natural frequencies, mode shapes, and transfer receptances (displacements per unit force) considering structural damping of an adhesively bonded single lap joint, and found that the resonant amplitudes are significantly reduced with increasing damping. Khalil and Kagho [12] showed that defects reduced the stiffness of the adhesively bonded single lap joint and resulted in a decrease in the resonant frequency and an increase in the damping capacity, using a simple vibration technique for the non-destructive assessment of the integrity of the adhesive joint.

Vaziri and Nayeb-Hashemi [13] investigated the effects of tubular joint geometry parameters and viscoelastic adhesive properties on the dynamic response of a tubular adhesive joint subjected to a harmonic axial load, and found that the system response was sensitive to the adhesive loss factor. Vaziri and Nayeb-Hashemi [14] presented a simple model to investigate the dynamic response of a repaired composite beam subjected to a harmonic peeling load. They considered both peeling and shear stresses in the viscoelastic adhesive layer, and predicted the first resonant frequency of the patched composite plates accurately compared with the experimental one.

Yuceoglu *et al*. [15–17] investigated the free vibration behavior of stiffened composite plates including an adhesive layer and presented a considerable number of studies in which they concentrated on the free, flexural vibrations of a relatively thick, orthotropic, composite base plate or panel reinforced by a stiffening plate strip located in any region of the composite plate. They showed that the material characteristics of the adhesive layer have a significant effect on the deformations of bonded multilayer composite plates and on their natural frequencies and mode shapes.

In order to join two composite or metal plates at a right angle, in practice, adhesive corner joints with single (SS) and double supports (DS), in which one or two plates are bent and bonded through an adhesive layer, are used. Apalak and Davies [18,19] indicated that increasing support length considerably reduced the stress concentrations around the adhesive-adherend free edges, and the adhesive fillet size affected these peak adhesive stress levels. Apalak *et al*. [20–22] also presented double containment corner joints, in which two plates are bonded in the slots of a containment support, and showed that a suitable support and slot depth can reduce the peak adhesive stresses around the free edges of the adhesive-plate interfaces.

Apalak *et al*. [23] investigated the three-dimensional free vibration behavior of an adhesively bonded corner joint with a SS and the effects of the geometrical parameters, such as support length, plate thickness, adhesive thickness, and joint length, on the natural frequencies, mode shapes, and modal strain energies of the adhesive joint, using both the finite element method and the back-propagation Artificial Neural Network (ANN) method. They found that the support length, plate thickness, and joint length all played important roles in the natural frequencies, mode shapes, and modal strain energies of the corner joint, whereas the adhesive thickness for the range of adhesive thickness studied had only a minor effect. They combined the Genetic Algorithm with the ANN models in order to determine the optimum geometrical dimensions which satisfied the maximum natural frequency and minimum modal strain energy conditions for each natural frequency and mode shape of the adhesively bonded corner joint.

In this study, the three-dimensional free vibration analysis of an adhesively bonded corner joint with double support was carried out. The couple effect of an additional horizontal support with the other geometrical parameters, such as the vertical support length, plate thickness, adhesive thickness, and joint length on the first ten natural frequencies and on the corresponding modal strain energies of the adhesive corner joint were also investigated. The optimal values of the geometrical design parameters were determined using the artificial neural networks and the genetic algorithm providing that the natural frequencies are maximized and the modal strain energies of the adhesive joint are minimized.

2. THE METHOD FOR MODE AND FREQUENCY ANALYSIS

In the case that the structure has constant stiffness, mass effects, and no time varying boundary conditions such as forces and displacements, the equation of motion for an undamped system is

$$[\mathbf{M}]\{\ddot{\mathbf{u}}\} + [\mathbf{K}]\{\mathbf{u}\} = \{0\}, \tag{1}$$

where $[\mathbf{M}]$ is the mass matrix and $[\mathbf{K}]$ is the stiffness matrix. The finite element method can be used for the dynamic analysis of the structures. The displacements within an element are interpolated from the element nodal degree of freedom (d.o.f.) $\{\mathbf{d}\}$ field

$$\{\mathbf{u}\} = [\mathbf{N}]\{\mathbf{d}\}, \tag{2}$$

where $[\mathbf{N}]$ is shape function matrix, and their first two time derivatives are written as

$$\{\dot{\mathbf{u}}\} = [\mathbf{N}]\{\dot{\mathbf{d}}\}, \quad \{\ddot{\mathbf{u}}\} = [\mathbf{N}]\{\mathbf{d}\}. \tag{3}$$

The shape functions $[\mathbf{N}]$ are functions of space only and nodal d.o.f. $\{\mathbf{d}\}$ are functions of time only. For an element of an undamped structure the equation of motion (Eq. (1)) is written as

$$[\mathbf{m}]\{\mathbf{d}\} + [\mathbf{k}]\{\dot{\mathbf{d}}\} = \{0\}, \tag{4}$$

where the element mass matrix

$$[\mathbf{m}] = \int_{V_e} \rho [\mathbf{N}]^T [\mathbf{N}] dV \tag{5}$$

and the element stiffness matrix

$$[\mathbf{k}] = \int_{V_e} [\mathbf{B}]^T [\mathbf{E}][\mathbf{B}] dV, \tag{6}$$

where

$$[\mathbf{B}] = [\partial][\mathbf{N}]. \tag{7}$$

For the assembled structure, the equation of motion (Eq. 1) becomes

$$[\mathbf{M}]\{\ddot{\mathbf{D}}\} + [\mathbf{K}]\{\mathbf{D}\} = \{0\}. \tag{8}$$

An undamped structure undergoes harmonic motion in which each d.o.f. moves in phase with all other d.o.f.:

$$\{\mathbf{D}\} = \{\overline{\mathbf{D}}\}\sin(\omega t) \quad \mathit{and} \quad \{\ddot{\mathbf{D}}\} = -\omega^2\{\overline{\mathbf{D}}\}\sin(\omega t), \tag{9}$$

where $\{\overline{\mathbf{D}}\}$ are amplitudes of nodal d.o.f vibration and ω is the circular frequency. Combining Eq. (9) with Eq. (8) yields

$$([\mathbf{K}] - \lambda[\mathbf{M}])\{\overline{\mathbf{D}}\} = \{0\}, \quad \mathit{where} \quad \lambda = \omega^2. \tag{10}$$

The solution of this eigenproblem

$$\det([\mathbf{K}] - \lambda[\mathbf{M}]) = \{0\}. \tag{11}$$

Each eigenvalue, λ_i, is associated with an eigenvector $\{\overline{\mathbf{D}}\}_i$ which is called a natural mode. The block Lanczos eigenvalue extraction method was used for the calculation of eigenvalues and eigenvectors since the models have large degree of freedoms [24,25]. The natural frequencies, f_i, rather than natural circular frequencies, ω_i, are given as

$$f_i = \frac{\omega_i}{2\pi}. \tag{12}$$

3. JOINT CONFIGURATION

This study investigates the effects of geometrical parameters, especially the horizontal support length, on the natural frequencies and mode shapes of an adhesively bonded corner joint with double support shown in Fig. 1. The supports are formed by bending the

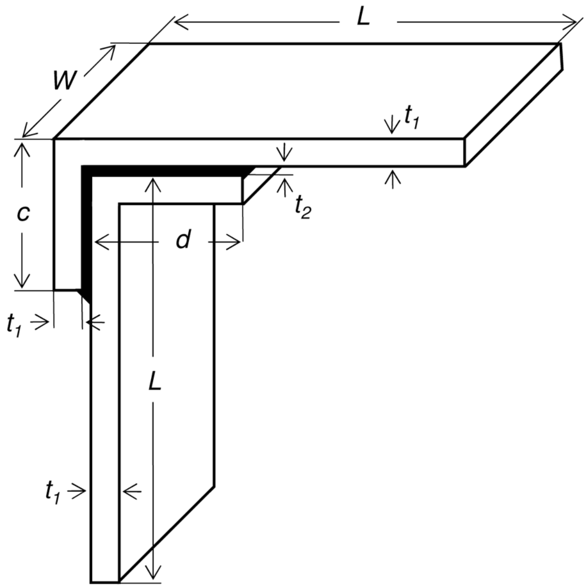


FIGURE 1 Geometry and dimensions of an adhesively bonded corner joint with DS.

horizontal and vertical plate ends such that the plates can be bonded through these support surfaces. The plates are made of aluminum and an epoxy adhesive is used to join the plates. The adhesive layer and plates were assumed to have linear elastic properties. The aluminum has a modulus of elasticity $E = 68.95\,GPa$, Poisson's ratio $\nu = 0.33$, and a density $\rho = 2770\,kg/m^3$ and the epoxy adhesive has a modulus of elasticity $E_{ep} = 3.33\,GPa$, Poisson's ratio $\nu_{ep} = 0.30$, and a density $\rho_{ep} = 1200\,kg/m^3$. The geometrical non-linearity and damping were not considered. The free vibration analysis was carried out to determine the effect of any geometrical parameter on the natural frequencies and the corresponding modal strain energies of the adhesive corner joint, while a joint length, L, of 120 mm, a joint width, W, of 25 mm, a plate thickness, t_1, of 2 mm, an adhesive thickness, t_2 of 0.2 mm, and vertical and horizontal support lengths, c and, d, of 20 mm were kept constant. The adhesively bonded corner joint with DS was fixed through the bottom surface of the vertical plate.

A three dimensional isoparametric structural finite element formulation was implemented to two plates and ax adhesive layer [24]. The finite element includes three degrees of freedom (displacements) at each node. The free edges of the vertical and horizontal adhesive

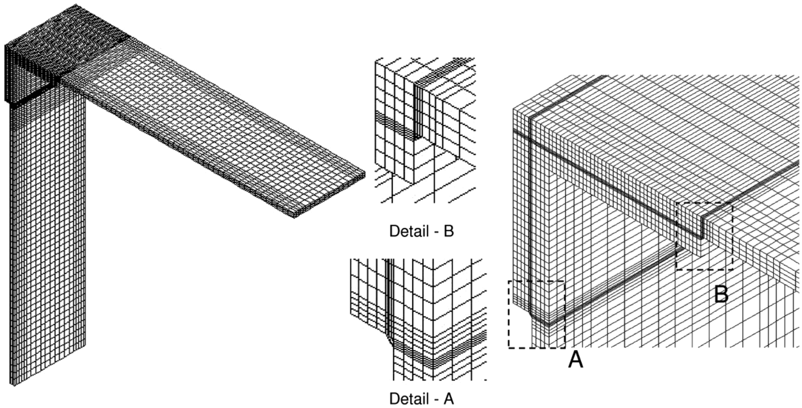


FIGURE 2 Mesh details of the adhesively bonded corner joint with DS.

layers, where the stress concentrations occurred, were refined in order to calculate accurately the strain energy in each element around these regions. The total degrees of freedom in the finite element model affects solution time during the eigenvalue extraction. The finite element model shown in Fig. 2 could achieve accurate results without causing errors and with a reasonable solution time. All calculations were performed in MATLAB environment [26].

4. FREE VIBRATION ANALYSIS

In this section the free vibration analysis was carried out for an adhesively bonded corner joint with DS, and the first ten natural frequencies of the adhesive joint were considered. The adhesive joint exhibits transverse (bending) and torsional modes for the first ten modes as shown in Fig. 3. The horizontal plate undergoes the bending modes at the natural frequencies of $\omega_1 = 36.3$, $\omega_2 = 74.0$, and $\omega_3 = 120.7$, whereas both horizontal and vertical plates experience bending modes at the natural frequencies of $\omega_4 = 505.8$, $\omega_6 = 908.7$, $\omega_8 = 1618$, and $\omega_{10} = 2402$. In addition, the vertical plate undergoes only torsional mode at $\omega_9 = 2145$ and the horizontal plate at $\omega_5 = 883.2$ and $\omega_7 = 1016$. The bending modes are dominant in the first ten modes of the adhesively bonded corner joint with DS.

Apalak *et al*. [23] investigated the effects of the mechanical and physical properties of the epoxy adhesive layer such as modulus of elasticity, Poisson's ratio, and density, on the natural frequencies and the corresponding modal strain energies of the adhesive corner joint with a single support. They showed that the adhesive modulus

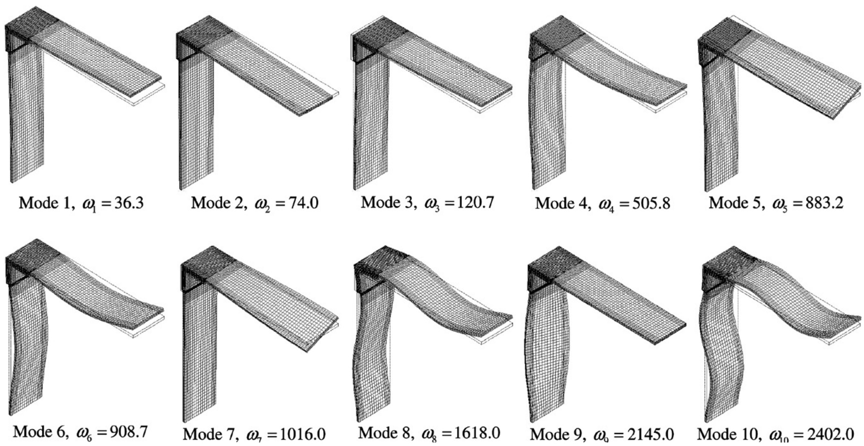


FIGURE 3 The first ten natural frequencies and mode shapes of the adhesively bonded corner with DS ($c = 25$, $d = 25$, $t_2 = 0.5$, and $t_1 = 5\,\text{mm}$).

had a minor effect on the adhesive/plate modulus ratios, E_{ep}/E, between 0.0005 and 1.0, and that this effect becomes negligible after an adhesive/adherend modulus ratio of 0.05, which corresponds to a modulus of 3.44 GPa of a typical epoxy adhesive. The adhesive Poisson's ratios between 0.2 and 0.4 also exhibited a minor effect, disappearing after a $\nu_{ep} = 0.3$ for a typical epoxy adhesive. In addition, the adhesive/plate density ratio (ρ_{ep}/ρ) between 0.050 and 1.0 had an insignificant effect since the adhesive layer is very thin.

The effects of the geometrical parameters, such as the vertical and horizontal support lengths, the adhesive thickness, the plate thickness, and the joint length, on the natural frequencies and the modal strain energies of the adhesive corner joint were also investigated. Increasing the vertical support length, c, from 15 to 60 mm causes increases of 6.3–21% in the natural frequencies of the bending modes, and increases of 1–26% in those of the torsional modes (Table 1). In addition, it causes increases of 14–47% in the modal strain energies of the bending modes and decreases of 18–28% in those of the torsional modes (not shown). The effect of the vertical support length is the same for the corner joints with single SS and DS supports (Table 1). The vertical support length contributes the total joint mass; therefore, the first ten natural frequencies increase. However, this effect is more evident for the bending modes, except for the first mode. In addition, the vertical plate of the adhesive corner joint becomes stiffer and the horizontal plate tends to deform, and the vertical plate exhibits an apparent effect on the modal strain energy of the corner joint. The horizontal support

TABLE 1 The Effect of the Vertical Support Length, c, on the First Ten Normalized Natural Frequencies of the Adhesively Bonded Corner Joints with a) SS [23] and b) DS (The Bold Value Indicates Peak Natural Frequency in Hz for Each Column)

	Normalized natural frequencies																			
	1		2		3		4		5		6		7		8		9		10	
Modes	DS	SS	DS	SS	DS	SS	DS	SS	DS	SS	DS	SS	DS	SS	DS	SS	DS	SS	DS	SS
c (mm)	**38.6**	**13.6**	**94.1**	**28.7**	**149.4**	**42.2**	**555.2**	**185.4**	**871.1**	**291.3**	**999.9**	**400.4**	**1036.3**	**518.8**	**1997.3**	**640.4**	**2755.8**	**766.5**	**2955.8**	**1394.6**
15	0.937	0.989	0.759	0.836	0.795	0.869	0.907	0.993	1.000	0.925	0.905	1.000	1.000	1.000	0.792	0.858	0.737	0.986	0.792	0.746
20	0.937	0.987	0.779	0.852	0.823	0.884	0.908	0.989	0.996	0.954	0.943	0.959	0.995	0.998	0.798	0.861	0.780	0.993	0.824	0.770
25	0.938	0.986	0.800	0.868	0.851	0.900	0.911	0.985	0.997	0.973	0.972	0.926	0.993	0.996	0.807	0.864	0.821	1.000	0.837	0.797
30	0.941	0.986	0.822	0.884	0.877	0.915	0.914	0.981	0.989	0.986	0.991	0.900	0.994	0.996	0.820	0.869	0.867	0.999	0.837	0.826
35	0.945	0.986	0.845	0.902	0.902	0.931	0.920	0.979	0.982	0.994	0.999	0.879	0.997	0.996	0.848	0.877	0.897	0.995	0.853	0.853
40	0.952	0.988	0.869	0.919	0.925	0.946	0.928	0.979	0.977	0.999	1.000	0.860	0.999	0.996	0.889	0.889	0.894	0.991	0.894	0.880
45	0.960	0.990	0.897	0.938	0.947	0.960	0.939	0.981	0.973	1.000	0.995	0.842	1.000	0.997	0.934	0.908	0.887	0.987	0.944	0.907
50	0.970	0.993	0.928	0.958	0.966	0.975	0.955	0.985	0.971	0.998	0.987	0.826	0.999	0.998	0.974	0.935	0.905	0.983	0.983	0.937
55	0.984	0.996	0.963	0.979	0.984	0.988	0.975	0.992	0.968	0.995	0.977	0.810	0.998	0.998	0.997	0.966	0.950	0.981	1.000	0.968
60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.961	0.994	0.968	0.794	0.996	0.996	1.000	1.000	1.000	0.982	0.996	1.000

contributes considerably to the natural frequencies of the corner joint; thus, a horizontal support causes 1.99–3.54 times as high natural frequencies and 2.52–6.59 times as high modal strain energies (DS, Table 1) in comparison with the corner joint with single support (SS, Table 1). The existence of a horizontal support results in both the natural frequencies and the corresponding modal strain energies.

As the horizontal support length, d, varies between 15 and 60 mm it causes variations of 1–25% in the bending natural frequencies (Table 2) and 2–41% in the modal strain energies (not shown). The effect of the horizontal support length is small for the lower bending modes but is apparent for the higher bending modes since the horizontal plate becomes stiffer. The torsional natural frequencies exhibit negligible variations (Table 2). The modal bending strain energies vary by 12–52% and become more evident in the higher modes due to the stiffer horizontal plate, whereas the torsional modal strain energies vary by 19–52% (not shown). A larger horizontal support length causes more elastic deformations through the horizontal plate rather than the adhesive layer since the corner joint becomes stiffer. Consequently, the bending natural frequencies and the modal strain energies increase. The mode shapes of the corner joint vary completely after a horizontal support length of 35 mm.

The adhesive thickness, t_2 between 0.1 and 0.5 mm indicated a minor effect (a minor increase of 1%) on both the first ten natural frequencies (Table 3) and the modal strain energies. Since the

TABLE 2 The Effect of the Horizontal Support Length, d, on the First Ten Normalized Natural Frequencies of the Adhesively Bonded Corner Joints with DS (The Bold Value Indicates Peak Natural Frequency in Hz for Each Column)

	Normalized natural frequencies									
Modes	1	2	3	4	5	6	7	8	9	10
d (mm)	**36.7**	**75.2**	**142.0**	**620.0**	**871.3**	**984.8**	**1558.5**	**2089.7**	**2209.0**	**2618.7**
15	1.000	1.000	0.838	0.807	1.000	0.922	0.590	0.753	1.000	0.936
20	0.992	0.997	0.870	0.804	0.996	0.960	0.619	0.758	1.000	0.948
25	0.983	0.994	0.899	0.805	0.997	0.984	0.654	0.767	1.000	0.952
30	0.974	0.990	0.925	0.809	0.996	0.997	0.691	0.787	1.000	0.949
35	0.965	0.985	0.948	0.819	0.992	1.000	0.731	0.820	1.000	0.943
40	0.954	0.978	0.968	0.834	0.985	0.996	0.776	0.867	0.998	0.938
45	0.944	0.969	0.984	0.859	0.977	0.990	0.825	0.921	0.995	0.939
50	0.932	0.958	0.994	0.894	0.971	0.983	0.878	0.971	0.992	0.950
55	0.921	0.946	0.998	0.941	0.971	0.981	0.936	1.000	0.991	0.971
60	0.909	0.936	1.000	1.000	0.979	0.986	1.000	0.998	0.993	1.000

TABLE 3 The Effect of the Adhesive Thickness, t_2, on the First Ten Normalized Natural Frequencies of the Adhesively Bonded Corner Joints with a) SS [23] and b) DS (The Bold Value Indicates Peak Natural Frequency in Hz for Each Column)

	Normalized natural frequencies																			
	1		2		3		4		5		6		7		8		9		10	
Modes	DS	SS	DS	SS	DS	SS	DS	SS	DS	SS	DS	SS	DS	SS	DS	SS	DS	SS	DS	SS
t_2 (mm)	**36.5**	**13.4**	**75.0**	**26.0**	**125.6**	**39.2**	**510.1**	**183.6**	**871.9**	**287.3**	**953.1**	**385.1**	**1035.9**	**518.1**	**1644.9**	**589.8**	**2212.4**	**766.7**	**2501.7**	**1195.0**
0.10	1.000	1.000	0.999	0.997	1.000	1.000	1.000	1.000	1.000	0.989	0.996	1.000	0.994	0.998	1.000	0.992	0.997	0.999	1.000	0.994
0.14	0.999	0.999	0.999	0.997	0.997	1.000	0.999	0.999	0.998	0.991	0.996	0.994	0.995	0.998	1.000	0.992	0.998	0.999	1.000	0.995
0.20	0.997	0.999	0.999	0.998	0.996	1.000	0.999	1.000	0.995	0.993	0.997	0.987	0.996	0.998	1.000	0.991	0.998	1.000	1.000	0.996
0.23	0.996	0.998	0.999	0.998	0.997	1.000	0.998	1.000	0.995	0.994	0.997	0.983	0.996	0.998	1.000	0.991	0.998	1.000	0.999	0.996
0.28	0.995	0.998	1.000	0.999	0.997	0.999	0.997	1.000	0.997	0.996	0.998	0.978	0.997	0.998	0.999	0.992	0.999	1.000	0.999	0.997
0.32	0.994	0.997	1.000	0.999	0.998	0.999	0.997	1.000	0.993	0.997	0.999	0.974	0.997	0.999	0.999	0.992	0.999	1.000	0.999	0.997
0.37	0.993	0.997	1.000	0.999	0.998	0.999	0.996	1.000	0.989	0.998	0.999	0.969	0.998	0.999	0.999	0.993	0.999	1.000	0.998	0.998
0.41	0.992	0.996	1.000	0.999	0.998	0.999	0.996	1.000	0.985	0.999	0.999	0.965	0.999	0.999	0.999	0.994	0.999	1.000	0.998	0.999
0.46	0.991	0.996	1.000	1.000	0.998	1.000	0.995	1.000	0.983	0.999	1.000	0.960	0.999	1.000	0.999	0.996	1.000	1.000	0.998	0.999
0.50	0.989	0.995	1.000	1.000	0.999	1.000	0.994	1.000	0.982	1.000	1.000	0.956	1.000	1.000	0.998	1.000	1.000	0.999	0.997	1.000

adhesive thickness is very much smaller than the plate thickness it can not contribute to the natural frequencies and the modal strain energies of the corner joint. This effect is observed in both the adhesive corner joints with SS and DS. However, the existence of a horizontal support results in 1.99–3.04 times as high natural frequencies and 5.35–9.53 times as high modal strain energies (DS, Table 3) in comparison with those of the corner joint with single support (SS, Table 3). However, the modal strain energy of the corner joint is desired to be reduced as much as possible.

The plate thickness, t_1, results in considerable increases in both the natural frequencies and the corresponding modal strain energies (Table 4). Thus, a plate thickness, t_1, between 1.5 and 5.0 mm results in increases of 62 and 74% in the natural frequencies of the bending modes and increases of 97% in the modal strain energies. The natural frequencies of the torsional modes increase by 45–74% and the modal strain energies by 34–97%. The effects of the plate thickness on both the natural frequencies and the modal strain energies are evident in comparison with the other geometrical parameters. The plate thickness exhibits the same effects for both corner joints with SS and DS (Table 4), whereas a horizontal support contributes considerably to the natural frequencies and the corresponding modal strain energies in comparison with the corner joint with a SS (Table 4).

The joint length, L, appears also to be a design parameter which reduces the first ten natural frequencies and corresponding modal strain energies. Thus, increasing the joint length, L, to between 100 and 320 mm causes a decrease of 96–98% in the natural frequencies and a decrease of 92–99% in the modal strain energies (Table 5). The corner joint becomes more flexible and slender with increasing joint length. Consequently, the natural frequencies and the modal strain energies decrease considerably. This effect is observed in both corner joints with SS and DS (Table 5). However, the horizontal support has the effect of increasing the natural frequencies and the modal strain energies.

As a result, the bending modes are dominant in the free vibration characteristics of the adhesively bonded corner joint with SS or DS. The horizontal and vertical support lengths, plate thickness, and joint length play an important role is the free vibration characteristics of the adhesive corner joint. The plate thickness causes significant increases in the natural frequencies and in the modal strain energies, whereas the joint length results in significant decreases. The existence of the horizontal support provides much higher natural frequencies as well as modal strain energies in comparison with the corner joint with a single support [23]. In practice, the design objective for the adhesive

TABLE 4 The Effect of the Plate Thickness, t_1, on the First Ten Normalized Natural Frequencies of the Adhesively Bonded Corner Joints with a) SS [23] and b) DS (The Bold Value Indicates Peak Natural Frequency in Hz for Each Column)

	Normalized natural frequencies																			
	1		2		3		4		5		6		7		8		9		10	
Modes	DS	SS	DS	SS	DS	SS	DS	SS	DS	SS	DS	SS	DS	SS	DS	SS	DS	SS	DS	SS
t_1 (mm)	**90.9**	**33.4**	**177.3**	**61.2**	**300.8**	**97.0**	**911.8**	**378.8**	**1258.2**	**456.4**	**2259.9**	**707.1**	**2364.5**	**1230.0**	**3973.0**	**1462.5**	**4925.6**	**1911.3**	**5766.0**	**2456.1**
1.5	0.273	0.213	0.291	0.216	0.286	0.204	0.380	0.241	0.555	0.341	0.307	0.384	0.350	0.203	0.265	0.162	0.305	0.196	0.287	0.246
2.0	0.377	0.325	0.397	0.335	0.391	0.318	0.530	0.389	0.665	0.533	0.384	0.414	0.391	0.316	0.375	0.280	0.417	0.308	0.395	0.387
2.3	0.435	0.387	0.456	0.401	0.450	0.381	0.613	0.464	0.690	0.642	0.443	0.399	0.450	0.383	0.435	0.347	0.477	0.373	0.456	0.463
2.7	0.516	0.475	0.537	0.491	0.531	0.471	0.727	0.571	0.687	0.769	0.526	0.415	0.532	0.475	0.518	0.440	0.560	0.463	0.542	0.567
3.1	0.596	0.563	0.617	0.580	0.611	0.560	0.841	0.685	0.679	0.775	0.609	0.514	0.615	0.566	0.601	0.533	0.639	0.552	0.626	0.668
3.4	0.677	0.650	0.697	0.667	0.690	0.648	0.965	0.802	0.700	0.779	0.690	0.616	0.696	0.657	0.683	0.626	0.715	0.641	0.707	0.766
3.8	0.758	0.738	0.774	0.753	0.768	0.736	0.992	0.913	0.753	0.776	0.771	0.714	0.776	0.745	0.764	0.720	0.789	0.730	0.785	0.859
4.2	0.839	0.825	0.850	0.837	0.846	0.824	0.991	0.988	0.827	0.801	0.849	0.811	0.854	0.831	0.845	0.815	0.862	0.820	0.860	0.939
4.6	0.920	0.913	0.926	0.920	0.924	0.912	0.995	1.000	0.912	0.901	0.926	0.905	0.928	0.916	0.923	0.909	0.932	0.910	0.932	0.990
5.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.982	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE 5 The Effect of the Joint Length, L, on the First Ten Normalized Natural Frequencies of the Adhesively Bonded Corner Joints with a) SS [23] and b) DS (The Bold Value Indicates Peak Natural Frequency in Hz for Each Column)

	Normalized natural frequencies																			
	1		2		3		4		5		6		7		8		9		10	
Modes	DS	SS	DS	SS	DS	SS	DS	SS	DS	SS	DS	SS	DS	SS	DS	SS	DS	SS	DS	SS
L (mm)	**52.3**	**37.4**	**110.8**	**76.5**	**188.2**	**113.6**	**733.5**	**510.0**	**1164.1**	**820.4**	**1335.8**	**889.7**	**1390.9**	**979.4**	**2386.3**	**1668.3**	**2849.3**	**2133.3**	**3621.5**	**2294.3**
100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
120	0.676	0.690	0.653	0.672	0.642	0.677	0.670	0.689	0.724	0.678	0.680	0.733	0.705	0.736	0.653	0.666	0.752	0.686	0.655	0.759
149	0.416	0.494	0.386	0.470	0.372	0.476	0.401	0.488	0.460	0.478	0.395	0.543	0.508	0.584	0.382	0.456	0.508	0.485	0.402	0.602
173	0.290	0.362	0.261	0.336	0.247	0.342	0.275	0.353	0.309	0.344	0.269	0.399	0.401	0.479	0.250	0.319	0.338	0.349	0.315	0.490
198	0.207	0.265	0.181	0.239	0.168	0.245	0.193	0.255	0.209	0.246	0.188	0.285	0.326	0.398	0.163	0.216	0.227	0.250	0.250	0.397
222	0.150	0.201	0.126	0.176	0.115	0.181	0.135	0.189	0.145	0.182	0.133	0.201	0.245	0.335	0.118	0.151	0.152	0.183	0.189	0.302
247	0.109	0.150	0.087	0.126	0.078	0.130	0.095	0.136	0.099	0.131	0.092	0.132	0.171	0.247	0.093	0.118	0.100	0.129	0.133	0.216
271	0.078	0.111	0.057	0.087	0.052	0.091	0.065	0.093	0.065	0.092	0.059	0.080	0.115	0.173	0.071	0.096	0.065	0.088	0.087	0.151
296	0.055	0.079	0.036	0.057	0.033	0.060	0.042	0.060	0.042	0.060	0.033	0.045	0.073	0.113	0.052	0.073	0.041	0.057	0.055	0.099
320	0.036	0.054	0.021	0.033	0.021	0.036	0.024	0.035	0.027	0.034	0.015	0.023	0.043	0.069	0.030	0.048	0.025	0.033	0.034	0.057

joints is to maximize the first natural frequency of the adhesive joint and to minimize the corresponding modal strain energy. The free vibration analysis indicated that the first natural frequency as well as the modal strain energy is increased with increasing support lengths and plate thickness whereas they are decreased with increasing joint length.

5. EFFECTS OF DESIGN PARAMETERS

An adhesively bonded corner joint with DS can be designed such that the natural frequencies are maximized and the corresponding modal strain energies are minimized. This optimization procedure requires a search among 6,626,804,580 models which were solved for the following design variables, such as a vertical support length $15, 15.1 \leq c \leq 60\,\text{mm}$, a horizontal support length $15, 15.1 \leq d \leq 60\,\text{mm}$, a plate thickness $1.5, 1.6 \leq t_1 \leq 5.0\,\text{mm}$, joint length $100, 101 \leq L \leq 320\,\text{mm}$ and an adhesive thickness $0.1, 0.2 \leq t_2 \leq 0.5\,\text{mm}$. In case the finite element method is used to determine the first ten natural frequencies and corresponding mode shapes of the corner joints, the extraction of the first ten eigenvalues becomes an expensive process depending on the number of the total degrees of freedom of the corner joint model. Then, the stress and strain distributions are calculated for each mode shape. This numerical method is not suitable for the direct search of the optimal design of the adhesive corner joint because it requires long computation time. However, the artificial neural networks (ANN) can predict promptly the values of the natural frequencies and corresponding modal strain energies for any set of design parameters [27]. This process does not need a long calculation time after a suitable model of the ANN is trained with necessary minimum input, output, and testing data. The input data are composed of the arbitrary values of the design variables whilst the output data includes the natural frequencies and corresponding strain energies for the values of these design variables. The testing data are used to determine the prediction ability of the trained neural network model.

The determination of the relationships among the plate thickness, the vertical and horizontal support lengths, and the joint length reduces the searching process for an optimal design satisfying a maximum natural frequency and a minimum modal strain energy conditions. Therefore, the trained ANN can be used to determine the effects of the design variable couples on the natural frequencies and on the modal strain energies. In this section, the first ten natural frequencies and corresponding modal strain energies of the adhesively bonded corner joint with double support are predicted using a proposed algorithm of the neural network model which needs input and testing data. For this purpose, the free vibration and stress analyses of the adhesive corner joint were carried out for random values between the lower and upper bounds of the design parameters whereas the mechanical properties of the plates and epoxy adhesive layer were kept constant. The first ten natural frequencies for a new set of design variables were extracted and the corresponding strain energies of the joint were calculated using the finite element method. The design variables and analysis results were collected in an input file used for the training seasons of the neural networks. The training seasons were repeated for each natural frequency and corresponding modal strain energy. The input of the neural network is composed of a random set of vertical and horizontal support lengths, plate thicknesses, joint lengths, and adhesive thicknesses and the output data includes either the natural frequency or the modal strain energy for each of the training seasons.

The structure of the proposed neural network, such as layer number, neuron number, and transfer functions, is designed depending on the prediction success of the ANN model using the testing data. This study uses the Levenberg-Marquardt (LM) back-propagation learning algorithm [27] in a feed-forward, two hidden layers network. The tangent sigmoid transfer function

$$f(x) = \frac{2}{1 + e^{-2x}} - 1 \tag{13}$$

and logarithmic sigmoid transfer function

$$f(x) = \frac{1}{1 + e^{-x}} \tag{14}$$

are used as the activation functions of the hidden layers and the output layer, respectively. The values of the training and test data were normalized to a range of 0–1. The LM back-propagation learning algorithm [27] was performed. A proposed neural network model was developed using the neural network toolbox of MATLAB [26]. The input data set includes 2,100 patterns based on the free vibration and stress analyses for a random set of design variables. The 2,000 patterns of the input data were used for the training of the neural network model and the 100 patterns selected randomly were used for testing the neural network model.

Figure 4 shows the layer configuration composed of 4, 10, 3, and 1 neurons in each layer (1–4) which is fast and predicts accurately the natural frequencies and strain energies. In the training phase, the

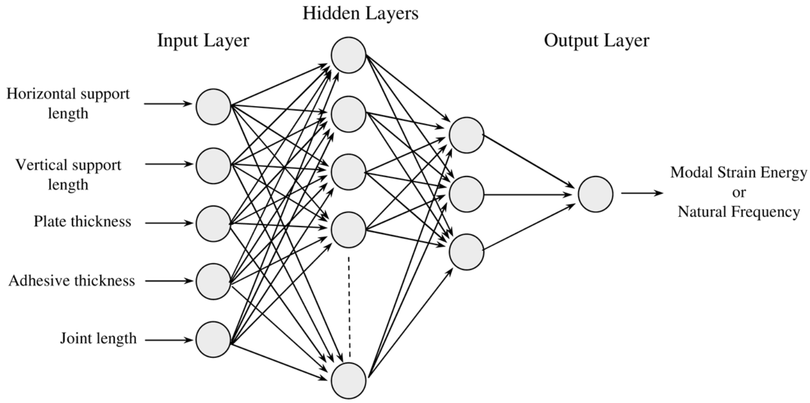
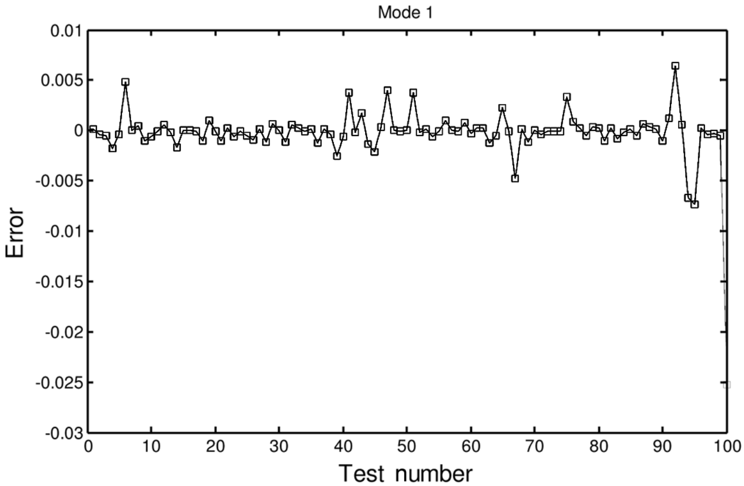


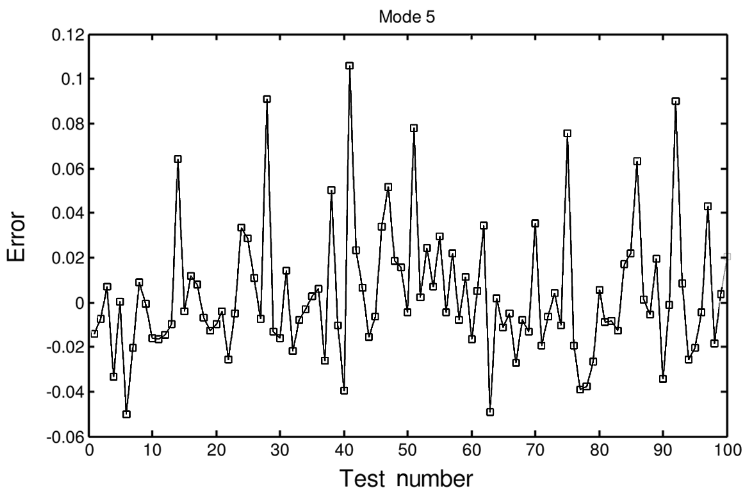
FIGURE 4 Architecture of an ANN model with two hidden layers.

performance function was selected as the mean square error (MSE) between the outputs predicted by the network and the test target outputs. The training procedure took a CPU time of 25–30 min. on a PIV (Pentium)® processor having a 3.0 Ghz CPU speed and 1 Gbyte RAM for a training cycle of 8,000 (epochs). The MSE of the training data could be reduced to an order of 10^{-8}–10^{-5} depending on the mode of the adhesive joint. Figure 5 shows the best and worst training sessions with percentage errors between the natural frequencies predicted by the ANN models and the testing data. Similar behaviors were also observed for the modal strain energies as a result of the ANN training phase, but not shown here. The neural network models were capable enough for predicting both natural frequencies and strain energies. Consequently, these well-trained ANN were used in order to determine the relationships among the design parameters and the couple effects of the design variables on the natural frequencies and corresponding modal strain energies. The natural frequencies were normalized with the maximum natural frequency value among the predicted output data for each mode, and the variations of the normalized natural frequency were compared for only the first mode of the adhesive corner joints with SS and DS in Figures 6–10.

Figure 6 shows the couple effect of the vertical support length ($15 \leq c \leq 60$ mm) and the joint length ($100 \leq L \leq 320$ mm) on the distributions of the first natural frequency. Increasing the joint length causes uniform decreases in the first natural frequency, whereas the vertical support length exhibits a minor effect. The joint length is more effective than the vertical support length. This effect becomes evident for a joint length smaller than 200 mm. The vertical support length



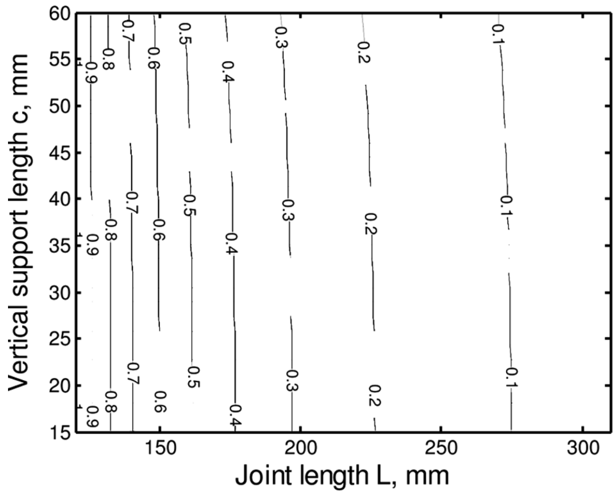
(a)



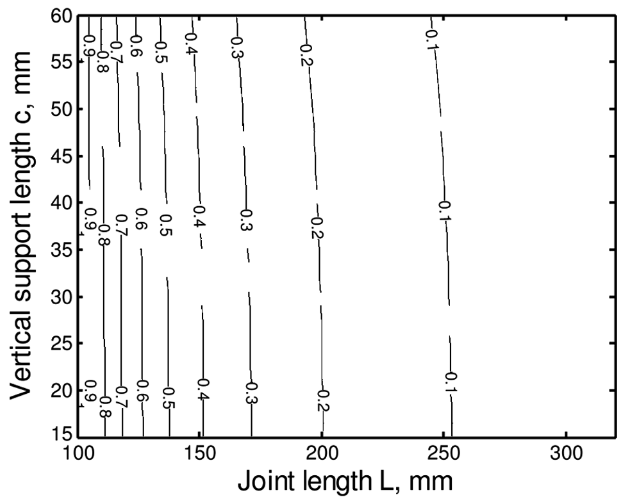
(b)

FIGURE 5 The percentage error levels in the predicted natural frequencies by ANN for 100 random test data: (a) the lowest error occurs in mode 1 and (b) the highest error occurs in mode 5 among the first 10 modes of the adhesively-bonded corner joint with DS.

and the joint length exhibit the same effects for both the adhesive corner joints with SS and DS joints. The existence of the horizontal support increases the natural frequencies of the corner joint and



(a)



(b)

FIGURE 6 The effects of the vertical support length, c, and the joint length, L, on the first normalized natural frequency of the adhesively bonded corner joint with a) SS [23] and b) DS.

causes a corner joint with a short joint length in comparison with the corner joint with a SS (Fig. 6a). Figure 7 shows that the joint length is a more dominant design parameter than the horizontal support length ($15 \leq d \leq 60$ mm) and increasing the joint length still has the effect of

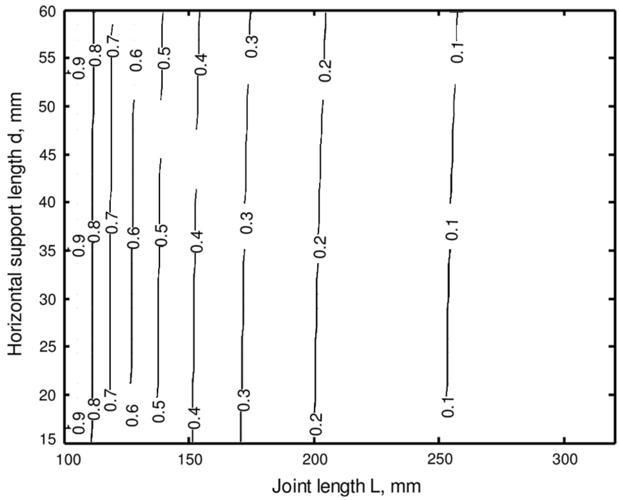
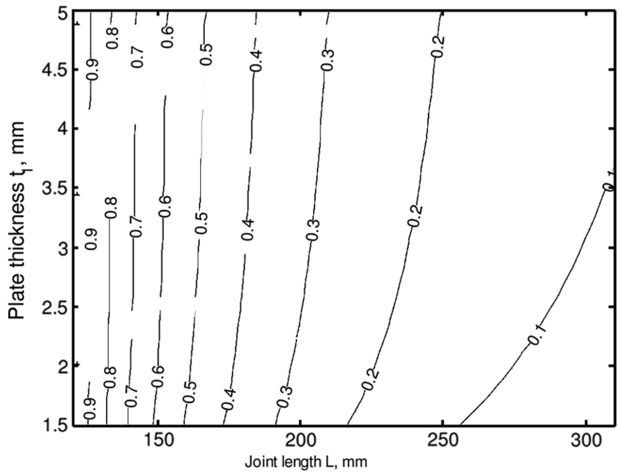


FIGURE 7 The effects of the horizontal support length, d, and the joint length, L, on the first normalized natural frequency of the adhesively bonded corner with DS.

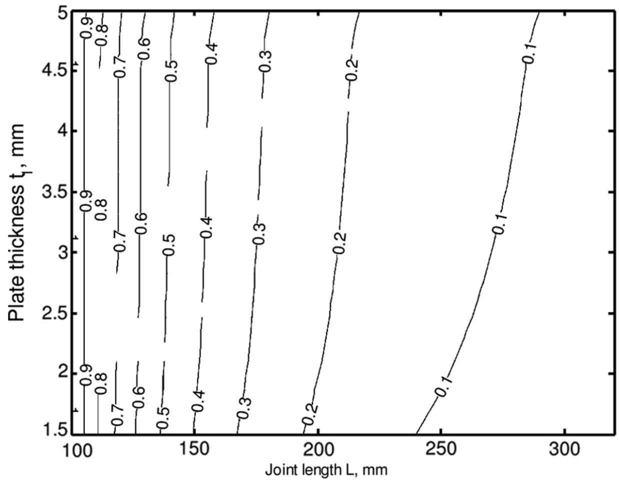
reducing the first natural frequency, whereas the horizontal support length results in small increases.

Figure 8 shows also the couple effects of the plate thickness ($1.5 \leq t_1 \leq 5\,\text{mm}$) and the joint length on the first natural frequency of the adhesive corner joint with SS and DS. The joint length is the dominant parameter, whereas the plate thickness results in small increases in the first natural frequency. Consequently, the joint length should be kept as small as possible in order to increase the first natural frequency. The horizontal support increases the first natural frequency and reduces the effect of the joint length; thus, a shorter joint length can be used in comparison with the corner joint with a single support (Fig. 8a).

The plate thickness, t_1, results in considerable increases in the first natural frequency in comparison with the vertical support length, c, and the horizontal support length, d, as shown in Figures 9 and 10, respectively. The plate thickness is also a dominant geometric parameter in increasing the natural frequencies. In addition, the modal strain energies of the adhesive corner joint were predicted using the trained neural networks for each mode and the effects of the geometric parameters on the modal strain energies were investigated. Similar results to those of the normalized natural frequencies were obtained. Finally, the joint length should be minimal and the plate thickness,



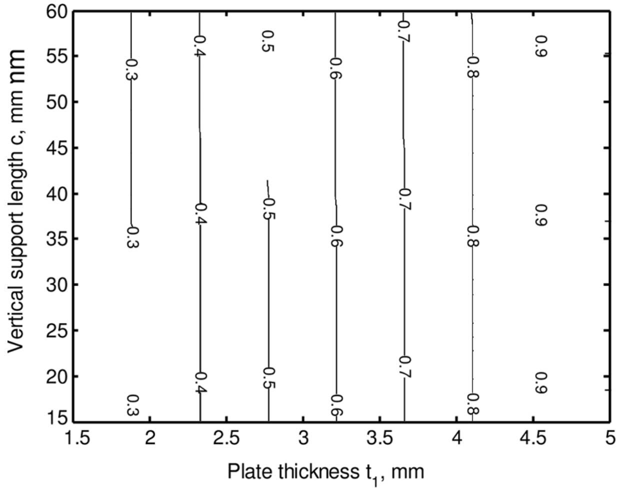
(a)



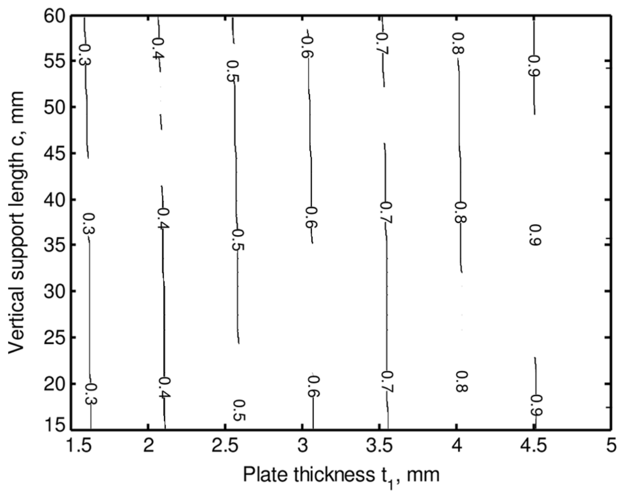
(b)

FIGURE 8 The effects of the plate thickness, t_1, and the joint length, L, on the first normalized natural frequencies of the adhesively bonded corner joint with a) SS [23] and b) DS.

which is more effective than the support lengths, should be as large as possible to obtain a joint design having maximum natural frequencies. In addition, the existence of the horizontal support length results in increases in the natural frequencies as well as in the modal strain energies.



(a)



(b)

FIGURE 9 The effects of the vertical support length, c, and the plate thickness, t_1, on the first normalized natural frequency of the adhesively bonded corner joint with a) SS [23] and b) DS.

6. OPTIMAL JOINT DESIGN

Genetic algorithms are a particular class of evolutionary algorithms that use techniques inspired by evolutionary biology such as inheritance, mutation, natural selection, and crossover. Genetic programming requires the evolution toward better solutions of a population

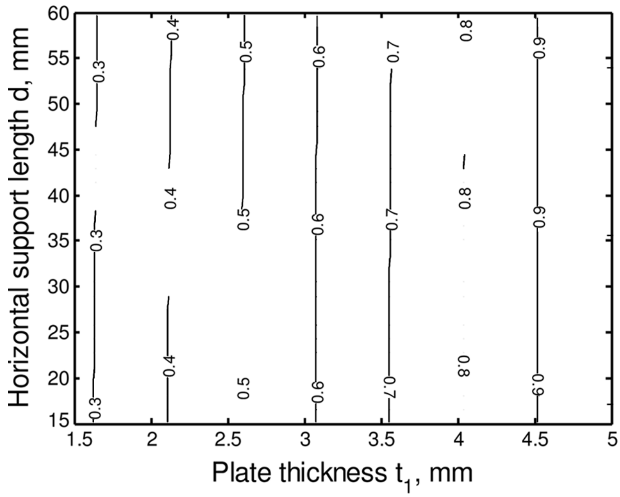


FIGURE 10 The effects of the horizontal support length, d, and the plate thickness, t_1, on the first normalized natural frequency of the adhesively bonded corner joint with DS.

of abstract representations (chromosomes) of candidate solutions (individuals) to an optimization problem [28–33]. The genetic algorithm can be used to find an optimal design of the adhesive corner joint with DS based on its free vibration characteristics. The genetic algorithm searches for an optimal joint design among random values of the design variables through a minimization process of an objective function in which the natural frequency is maximized and the modal strain energy is minimized. The trained neural networks were used to predict the values of the fitness function for the random values of the design variables generated by the genetic algorithm. The genetic algorithm toolbox of MATLAB [26] was used to solve the optimization problem based on the free vibration characteristics of the adhesively bonded corner joint, which can be defined in the following manner.

The objective function: find vertical support length, c, horizontal support length, d, plate thickness, t_1, and joint length, L, for each mode by maximizing

$$f(\omega_i, U_i) = c_1 \times \omega_i - c_2 \times U_i,$$

where $c_1 = c_2 = 0.5$, ω_i is the ith natural frequency, and U_i is the corresponding modal strain energy,
subject to
$15 \leq c \leq 60\,\text{mm},\ 1.5 \leq t_1 \leq 5.0\,\text{mm},\ 15 \leq d \leq 60\,\text{mm},$ and $100 \leq L \leq 320\,\text{mm}.$

Apalak *et al.* [23] showed that the natural frequencies and modal strain energies of the adhesive corner joint are not affected by the mechanical properties and thickness of the adhesive layer. Consequently, the searching process for the optimal joint design can be reduced by neglecting the mechanical properties and the thickness of the adhesive layer. The proposed neural networks were developed based on the natural frequency and modal strain energy for each mode of the adhesive joint, and were coupled with the genetic algorithm which predicts the optimal horizontal, vertical support lengths, plate thickness, and joint length.

Tables 6–9 compare the optimal joint dimensions for the first ten modes of the adhesive corner joint with DS with those of the corner joint with SS [23], whilst one of the geometrical parameters is changed in a specified range. The horizontal support length, d, becomes minimal for the first two modes in which only the horizontal plate deforms as the vertical support length is increased (Table 6), whereas a maximal horizontal support length is required for the higher modes in which the vertical plate also deforms. In addition, the plate thickness, t_1, and the joint length, L, become always minimal. In case a horizontal support is used, the plate thickness, t_1, becomes minimal for all modes in comparison with the adhesive corner joint with a SS (Table 6). Similar results can be observed as the horizontal support length is increased (Table 7). Thus, the plate thickness and the joint length become smaller and the vertical support length also becomes minimal for only the lower modes whilst it is maximal for the higher modes.

As the plate thickness is increased all geometrical parameters become minimal for the first mode (Table 8). However, a reverse relation appears between both support lengths and the plate thickness for the higher modes such that both support lengths decrease uniformly from the upper limit as the plate thickness increases. In addition, the joint length is still minimal. The plate thickness has more of an effect on the natural frequencies and the corresponding modal strain energies than the horizontal and vertical support lengths. Increasing the joint length requires using a larger horizontal vertical support length and plate thickness for all modes of two adhesive corner joints (Table 9). Finally, the plate thickness and joint length have more of an effect on the vibration characteristics of an optimal design of the adhesive corner joint with DS, and the use of a horizontal support increases the natural frequencies as well as the the modal strain energies. In addition, these vibration characteristics can be improved by increasing the horizontal and vertical support lengths when the joint length or the plate thickness adversely affect the vibration characteristics.

TABLE 6 The Optimum Horizontal Support Lengths, d, Plate Thicknesses, t_1, and Joint Lengths, L, for Various Vertical Support Lengths, c, for Each Mode of the Adhesively Bonded Corner Joint with a) SS [23] and b) DS

	Mode 1					Mode 2					Mode 3					Mode 4					Mode 5				
	DS			SS		DS			SS		DS			SS		DS			SS		DS			SS	
c (mm)	d	t_1	L	t_1	L	d	t_1	L	t_1	L	d	t_1	L	t_1	L	d	t_1	L	t_1	L	d	t_1	L	t_1	L
15	15	1.5	120	3.5	120	15	1.5	120	3.2	120	45	1.5	120	2.7	120	60	1.5	120	3.2	120	60	1.5	120	2.3	120
20	15	1.5	120	3.4	120	15	1.5	120	3.1	120	45	1.5	120	2.8	120	60	1.5	120	3.1	120	60	1.5	120	2.3	120
30	15	1.5	120	3.2	120	15	1.5	120	3.0	120	46	1.5	120	2.8	120	60	1.5	120	2.9	120	26	1.5	120	2.3	120
40	15	1.5	120	3.1	120	15	1.5	120	3.0	120	48	1.5	120	2.8	120	60	1.5	120	2.7	120	29	1.5	120	2.2	120
50	15	1.5	120	3.0	120	15	1.5	120	2.9	120	49	1.5	120	2.9	120	60	1.5	120	2.5	120	15	1.5	120	2.2	120
60	15	1.5	120	2.9	120	15	1.5	120	2.8	120	50	1.5	120	2.9	120	60	1.5	120	2.4	120	15	1.5	120	2.2	120

	Mode 6					Mode 7					Mode 8					Mode 9					Mode 10				
	DS			SS		DS			SS		DS			SS		DS			SS		DS			SS	
c (mm)	d	t_1	L	t_1	L	d	t_1	L	t_1	L	d	t_1	L	t_1	L	d	t_1	L	t_1	L	d	t_1	L	t_1	L
15	15	1.5	120	2.1	120	60	1.5	120	3.9	120	45	1.5	120	28	120	51	1.5	120	5.0	120	60	1.5	120	4.9	142
20	60	1.5	120	2.1	120	60	1.5	120	3.9	120	60	1.5	120	27	120	46	1.5	120	5.0	120	60	1.5	120	4.4	132
30	47	1.5	120	2.8	120	60	1.5	120	3.9	120	60	1.5	120	29	120	28	1.5	120	5.0	120	60	1.5	120	4.7	120
40	38	1.5	120	2.9	120	60	1.5	120	4.0	120	60	1.5	120	29	120	20	1.5	120	4.0	120	60	1.5	120	4.4	120
50	38	1.5	120	3.5	120	60	1.5	120	4.0	120	59	1.5	120	31	120	43	1.5	120	3.4	120	60	1.5	120	3.8	120
60	60	1.5	120	2.8	120	60	1.5	120	4.0	120	52	1.5	120	37	120	34	1.6	120	3.4	129	60	1.5	120	4.2	120

TABLE 7 The Optimum Vertical Support Lengths, c, Plate Thicknesses, t_1, and Joint Lengths, L, for Various Horizontal Support Lengths, d, for Each Mode of the Adhesively Bonded Corner Joint with DS

	Mode 1			Mode 2			Mode 3			Mode 4			Mode 5			Mode 6			Mode 7			Mode 8			Mode 9			Mode 10		
d (mm)	c	t_1	L	c	T_1	L	c	t_1	L	c	t_1	L	c	t_1	L	c	t_1	L	c	t_1	L	c	t_1	L	c	t_1	L	c	t_1	L
15	15	1.5	120	15	1.5	120	45	1.5	120	60	1.5	120	15	1.5	120	60	1.5	120	60	1.5	120	52	1.5	120	35	1.5	120	60	1.5	120
25	15	1.5	120	15	1.5	120	49	1.5	120	60	1.5	120	15	1.5	120	60	1.5	120	60	1.5	120	51	1.5	120	51	1.5	120	60	1.5	120
30	15	1.5	120	15	1.5	120	49	1.5	120	60	1.5	120	60	1.5	120	60	1.5	120	60	1.5	120	59	1.5	120	42	1.5	120	60	1.5	120
40	15	1.5	120	15	1.5	120	49	1.5	120	60	1.5	120	60	1.5	120	60	1.5	120	60	1.5	120	59	1.5	120	39	1.5	120	60	1.5	120
50	15	1.5	120	15	1.5	120	49	1.5	120	60	1.5	120	60	1.5	120	60	1.5	120	60	1.5	120	59	1.5	120	51	1.5	120	60	1.5	120
60	15	1.5	120	15	1.5	120	49	1.5	120	60	1.5	120	60	1.5	120	60	1.5	120	60	1.5	120	59	1.5	120	54	1.5	120	60	1.5	120

TABLE 8 The Optimum Vertical Support Lengths, c, Horizontal Support Lengths, d, and Joint Lengths, L, for Various Plate Thicknesses, t_1, for Each Mode of the Adhesively Bonded Corner Joint with a) SS [23] and b) DS

	Mode 1					Mode 2					Mode 3					Mode 4					Mode 5				
	DS			SS		DS			SS		DS			SS		DS			SS		DS			SS	
t_1 (mm)	c	d	L	c	L	c	d	L	c	L	c	d	L	c	L	c	d	L	c	L	c	d	L	c	L
1.5	15	15	120	60	120	58	58	120	60	120	57	57	120	59	120	60	60	120	15	120	24	24	120	20	120
2.0	15	15	120	60	120	55	55	120	60	120	57	57	120	60	120	60	60	120	15	120	15	15	120	30	120
3.0	15	15	120	15	120	38	38	120	60	120	49	49	120	60	120	32	32	120	15	120	59	59	120	15	120
4.0	15	15	120	15	120	15	15	120	15	120	37	37	120	60	120	15	15	136	15	143	55	55	120	15	120
5.0	15	15	124	15	129	15	15	137	15	141	26	26	120	60	171	15	15	157	15	170	55	55	129	15	124
	Mode 6					Mode 7					Mode 8					Mode 9					Mode 10				
	DS			SS		DS			SS		DS			SS		DS			SS		DS			SS	
t_1 (mm)	c	d	L	c	L	c	d	L	c	L	c	d	L	c	L	c	d	L	c	L	c	d	L	c	L
1.5	38	38	120	60	120	60	60	120	15	120	59	59	120	55	120	48	48	120	56	120	60	60	120	47	120
2.0	60	60	120	15	120	60	60	120	60	123	53	53	120	55	120	48	48	120	17	120	60	60	120	47	120
3.0	60	60	120	42	120	60	60	120	15	120	50	50	120	57	120	39	39	120	21	120	60	60	120	43	120
4.0	36	36	120	52	120	60	60	120	55	120	49	49	120	60	120	37	37	120	25	120	52	52	120	40	120
5.0	36	36	122	60	128	52	52	120	40	120	41	41	120	17	141	32	32	120	29	120	28	28	167	35	121

TABLE 9 The Optimum Vertical Support Lengths, c, Horizontal Support Lengths, d, and Plate Thicknesses, t_1 for Various Joint Lengths, L, for Each Mode of the Adhesively Bonded Corner Joint with a) SS [23] and b) DS

	Mode 1					Mode 2					Mode 3					Mode 4					Mode 5				
	DS			SS		DS			SS		DS			SS		DS			SS		DS			SS	
L (mm)	c	d	t_1	c	t_1	c	d	t_1	c	t_1	c	d	t_1	c	t_1	c	d	t_1	c	t_1	c	d	t_1	c	t_1
120	15	15	4.1	15	3.5	49	49	3.2	60	2.8	60	60	3.2	60	2.9	57	57	2.7	15	3.2	60	60	4.3	28	2.3
140	15	15	4.5	15	3.7	53	53	3.5	60	3.1	60	60	3.6	60	3.1	35	35	3.4	15	3.4	60	60	4.2	29	2.3
160	15	15	4.8	15	4.0	54	54	3.9	60	3.4	60	60	4.1	60	3.3	52	52	3.2	15	3.6	60	60	4.2	41	2.3
180	15	15	5.0	15	4.3	53	53	4.2	60	3.7	60	60	4.4	60	3.5	17	17	3.9	15	3.8	60	60	4.4	15	2.6
220	15	15	5.0	15	4.7	60	60	4.8	60	4.1	60	60	5.0	60	3.8	60	60	3.5	15	3.9	60	60	4.9	15	5.0
260	15	15	5.0	15	5.0	60	60	5.0	60	4.6	60	60	5.0	60	4.2	60	60	3.5	15	4.0	60	60	5.0	15	5.0
300	15	15	5.0	15	5.0	60	60	5.0	60	5.0	60	60	5.0	60	4.5	15	15	4.3	15	4.2	37	37	5.0	15	4.8

	Mode 6					Mode 7					Mode 8					Mode 9					Mode 10				
	DS			SS		DS			SS		DS			SS		DS			SS		DS			SS	
L (mm)	c	d	t_1	c	t_1	c	d	t_1	c	t_1	c	d	t_1	c	t_1	c	d	t_1	c	t_1	c	d	t_1	c	t_1
120	34	34	3.5	48	3.5	60	60	5.0	55	4.0	60	60	4.0	57	3.5	60	60	3.6	29	5.0	37	37	4.5	60	4.2
140	39	39	3.7	53	3.6	60	60	5.0	15	4.0	59	59	4.3	60	3.4	30	30	5.0	60	3.6	43	43	4.3	39	4.5
160	46	46	3.9	58	3.8	60	60	5.0	54	3.9	60	60	4.8	60	3.3	30	30	3.6	48	4.9	43	43	4.1	39	4.2
180	54	54	4.1	60	4.0	60	60	5.0	55	3.9	60	60	5.0	55	3.5	36	36	3.6	49	5.0	44	44	3.9	58	3.6
220	60	60	4.8	60	4.3	22	22	5.0	60	3.9	60	60	5.0	60	3.8	25	25	5.0	60	5.0	15	15	4.0	15	4.2
260	60	60	5.0	60	4.7	60	60	3.9	15	2.6	57	57	5.0	15	5.0	32	32	5.0	60	5.0	49	49	3.8	60	3.9
300	60	60	5.0	60	5.0	60	60	4.1	15	2.6	60	60	5.0	60	5.0	42	42	5.0	60	5.0	58	58	4.3	60	4.0

8. CONCLUSIONS

The parameters affecting the optimal design of an adhesively bonded corner joint with double support were determined based on a three-dimensional free vibration analysis. The adhesive joint undergoes mostly bending modes for its first ten natural frequencies. The natural frequencies and corresponding modal strain energies decrease with increasing joint length, whereas the horizontal and vertical support lengths and particularly the plate thickness cause considerable increases in both natural frequencies and modal strain energies. The existence of the horizontal support causes increases in both natural frequencies and modal strain energies in comparison with the corner joint with a SS [23]. The genetic algorithm and the ANN were implemented successfully for determining optimal joint dimensions for each mode. The plate thickness is the dominant design parameter which maximizes the natural frequencies and minimizes the strain energies, whereas the joint length has an adverse effect. The vibration characteristics can be also improved by increasing the horizontal and vertical support lengths when the joint length or the plate thickness adversely affect the vibration characteristics.

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